Constrained modifications of non-manifold b-reps

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ABSTRACT

Non-manifold boundary representations (b-reps) are increasingly used in Geosciences for a variety of applications (3D geographical information systems, basin modeling, geophysical processing, etc.). Meanwhile, the uncertainties associated with subsurface data make it desirable to modify such models efficiently. We present a method to deform locally a surface in a triangulated b-rep while maintaining a constant number of spatial regions in the model. This method does not require completely rebuilding the model, and thus allows efficient and robust updates of the model definition. The method requires that the reshaped surface does not intersect the boundaries of its adjoining regions, which can be checked using existing collision detection algorithms. Also, the non-manifold contacts must be updated after the modification, and the triangles must be altered, to maintain sealed regions. For this, we propose to parameterize locally the surfaces that the modified surface moves along. This parameteric space is used to (1) constrain the displacement of the deformed surface border and, 2) re-triangulate in the plane the neighboring surfaces around the modified contacts. The method, tested in the context of an interactive graphical interface, allows efficient and robust updates of the model definition. The resulting solids may not belong to the same parameteric family as before, as faces may appear or disappear during the modification. This paper is organized as follows.

1. INTRODUCTION

For about three decades, boundary representations (b-reps) [26, 20, 11] have been primarily used in the design of mechanical parts. In these applications, the models are usually not built once and for all, but undergo many modifications to account for new design constraints [31]. Therefore, two complementary approaches to b-rep modification using model parameters have been developed:

- Direct b-rep modification using variational modeling [16] consists in applying relatively small perturbations while maintaining relationships between the interfaces.
- Indirect b-rep modifications are performed on other volume representations, such as constructive solid geometry (CSG), in which the geometric primitives can be manipulated through their size, position, aspect ratio, . . . , or feature-based modeling [30, 27], in which more advanced modifications can be performed using constraint-based editing of high-level features. In both cases, the b-rep is re-evaluated after the modification of the alternate representation [24].

In Geosciences, which is another important application domain of b-reps, a need for model modification also exists. However, the model parameters are usually not available directly in earth models. Instead, the volume boundaries consist of free-form surfaces, usually triangulated, which have been constrained to honor available subsurface data [9, 19, 13]. Conversion to CSG or to feature-based representation seems therefore inadequate for modifying such models.

As for the representation itself, "almost all man-made solids" can be represented using manifold structures [28]. However, earth models often have to account for non-manifold singularities at the contact between multiple geological interfaces, for instance between faults and horizons [9, 1, 13] (Figure 1).

In this paper, we introduce a technique for modifying interactively an interface in such a non-manifold model without paying the cost of a complete model rebuild (Figure 1). This modification is constrained so that the number of volume regions remains the same during the modification. The resulting solids may not belong to the same parameteric family [31] as before, as faces may appear or disappear during the modification. This paper is organized as follows. First, the motivation of this work is explained, and related works are described. The modification method is then presented, consisting in three main steps: first, classification and parameterization of the fixed interfaces (Section 3); second, constrained deformation (Section 4); third, topological surgery to update the b-rep definition (Section 5).
2. MOTIVATION AND RELATED WORK

2.1 Volume modeling in Geosciences

3D modeling of the Earth’s interior is currently used in many application areas, e.g., oil and gas exploration and production, mining and water resource assessment, civil engineering. Indeed, subsurface models make it possible to integrate acquisition data—geophysical signal, borehole information—in a unified representation that can be used for a variety of purposes. Yet, the accurate modeling of geological structures has long been identified as a difficult task [32, 19]. The challenge comes mostly from:

1. The complex structures to represent: faulted blocks, salt domes, complex folds, etc.
2. The variable precision and resolution of subsurface data, due to measurement and processing techniques.
3. The uncertainty associated to the model itself, which is always a mere approximation of the reality.

As they provide the required expressive power for handling geological complexity, non-manifold b-reps appear to be the most interesting representation for subsurface modeling [10, 22, 9, 1, 19, 13]. In practice, the geological interfaces are first created from the available subsurface data as manifold triangulated surfaces [19]. Intersections are then computed between these interfaces, creating new surface boundaries at the intersections, and radial edges [33] are set around these intersections to carry an explicit representation of the non-manifold topology. From this information, volume regions can be determined using a simple graph traversal algorithm. Note that the radial edges are only needed along the border of the manifold surfaces. Therefore, most b-reps used in Geosciences [13, 4] make a clear distinction between the micro-topology, which concerns the manifold description of triangulated surfaces, and the non-manifold macro-topology, which describes how these manifold patches are connected to each other along their borders.

The items 2 and 3 above require not only to be able to build a 3D b-rep, but also to modify it conveniently. Such modifications may be used for time-to-depth conversion in geophysical processing [10, 34, 6] or for generating several equiprobable geometric realizations that honor the data [14]. In this work, we have tackled more specifically the demanding aspects of interactive expert editing. Such editing may be needed away from data, to correct the unanticipated behavior of algorithms, or to tune the parameters of geophysical inversion [4].

2.2 Geological Validity

Whatever the application, the representational validity of a model must not be infringed by a modification. Thus, the non-intersection condition [20, 28] has to be honored: two volumes may only intersect at their common faces, two faces at their common edges, and two edges at their common vertices. For this, a reliable algorithm for computing the intersection between two triangulated surfaces is needed, which is complicated because of the limited precision of numbers on computers. The reader is referred to [23, 28] for further developments on this subject.

Not less important than topological and geometrical validity, the geological consistency has to be maintained throughout the modification. There can be several meanings for geological consistency. We will give here a minimal definition that can easily be checked using topological information, provided that the model has a valid representation. Let \( M \) denote a b-rep volume model (Figure 2) made of a domain boundary \( S_B \), a set of \( m \) fault surfaces \( \{F_1, \cdots, F_m\} \), and a set of \( n \) layer boundaries \( \{H_1, \cdots, H_n\} \). A free border is a manifold surface border which is not bound to another surface by radial edges [33]. A region of the model \( M \) is a closed volume bounded by possibly non-manifold interfaces. A layer is a collection of regions that have the same genetic meaning. As a consequence, a region can only belong to a single layer.

From these definitions, two fundamental geological validity conditions for \( M \) can be formulated:

1. The surfaces \( S_B \) and \( \{H_1, \cdots, H_n\} \) cannot have any
free border. Indeed, only a fault can have free borders, which correspond to a null displacement of blocks.

2. Any two layer boundaries $H_i$ and $H_j$ cannot cross each other: by definition, one of the oriented faces of a layer boundary identifies one and only one layer; thus, if $H_i$ and $H_j$ could cross each other, a region of space would belong to two layers, which is impossible.

Of course, geological validity can also involve more advanced criteria based on mass conservation, deformation and rupture mechanisms, ... [25]. In this work, only the minimal definition formulated above is respected, as such additional criteria may differ according to the geological context.

2.3 Related work

Several approaches for modifying triangulated non-manifolds have been described previously. One of the most well-known methods is probably free-form deformation (FFD) [29] which can be generically used to deform geometric models. The method uses a smooth spatial deformation function, such as a tri-cubic spline, which is defined on a 3D control mesh. An approximation of the FFD is usually achieved by applying the deformation to the vertices of the model. Due to this discrete approximation, intersections between faces of a b-rep may appear if the FFD is not linear. When such interferences are detected, the mesh can be refined to yield a better FFD approximation. Based on this paradigm, several approaches have been defined to directly modify geometric models [12, 3]. In [12], an inverse method is used to compute the displacement of the control mesh from the desired displacement of some points of the model. In Simple Constrained deformations (SCODEF) [3], the elements inside an influence sphere are moved according to a radial function (Figure 3). With regard to the two validity conditions (Section 2.2), these spatial deformation methods can give good results, if discretization problems are carefully handled. In fact, these approaches keep to a condition of constant topology through the modification. However, for many applications, this condition is too restrictive, and it is desirable to leave room for controlled topological changes.

For this purpose, parametric modeling techniques [30, 27, 24] would be interesting. Yet, a proper parametric definition is hard to find for geological models, as they are created from unstructured and heterogeneous data [19]. Besides, a variational parametric family [31] implies that the modified model is homeomorphic to the initial model. This condition is too restrictive for some modifications, as geologically valid modifications may change the topology.

In [10], a custom b-rep representation that consists of free-form surfaces is described. As these surfaces can be modified independently, a large range of modifications is possible.

Yet, this representation uses a volatile definition of the regions, as the intersections between the interfaces are not stored. Thus, no mechanism exists to maintain the persistence of regions through a modification. This is undesirable, as regions may contain important information such as analytical descriptions of subsurface properties.

In [6], an interface can be removed from the model, globally modified under geometric constraints, and reincorporated into the volume model using a surface “cut with constraints” operation that honors contacts. For the user, this operation is quite flexible, but it requires a complete model reconstruction, as the modifications may imply drastic changes in the model definition. This process is computationally expensive, and does not guarantee the persistence of geological invariants (number of regions, definition of layers, etc.).

2.4 Outline of the proposed method

We now present a method suitable for locally modifying an existing valid model under constraints, which consists in deforming the interfaces separately. In what follows, we describe how to locally deform an interface $\mathcal{I}_d$ that bounds two regions $R_1$ and $R_2$. Preserving the model validity requires to keep constant the number and relative layout of regions. Let $\mathcal{I}_d$ denote the slip interface defined by: $\mathcal{I}_d = (\partial R_1 \cup \partial R_2) \setminus \mathcal{I}_d$. Then, model validity is ensured by the following constraints:

Constraint 1 the borders of $\mathcal{I}_d$ can only be displaced continuously along the slip surface $\mathcal{I}_d$.

Constraint 2 $\mathcal{I}_d$ may only move inside $R_1 \cup R_2$.

Based on these constraints, we now describe how the modification can be implemented in the frame of a graphical manipulator, which specifically deforms a non-manifold contact between two surfaces. Section 3 describes the operations needed for initializing this manipulator. Then, a method to ensure Constraint 1 during the displacement is proposed (Section 4). In Section 5, the topological updates performed at the end of the manipulation are described.

3. CLASSIFICATION OF SURFACES

For real-time interaction, the modification is performed locally. Therefore, the connected set of cells inside a user-defined sphere of influence is first selected. Among these cells, several subsets are created (Figure 4):

- $\mathcal{C}_{\mathcal{I}_d \cap \mathcal{I}_f}$ makes up the modifiable contact between $\mathcal{I}_f$ and $\mathcal{I}_d$. This contact comprises the radial edges binding the border triangles of $\mathcal{I}_d$ to the border triangles of $\mathcal{I}_f$. During the modification, this contact must remain on the slip surface $\mathcal{I}_d$; its topology should not be modified.

- $\mathcal{E}_{\partial \mathcal{I}_d}$ and $\mathcal{E}_{\partial \mathcal{I}_f}$ consist of the other border edges of both surfaces. As these elements make up the junction of the modified domain with the remainder of the model, their geometry and topology will be fixed during the modification.

- $\mathcal{N}_{\mathcal{I}_d}$ and $\mathcal{N}_{\mathcal{I}_f}$ consist of, respectively, the interior nodes and triangles of the interface $\mathcal{I}_d$. By definition, the nodes $\mathcal{N}_{\mathcal{I}_d}$ may be displaced during the modification. Their connectivities may be modified or not during the deformation of $\mathcal{I}_d$.

- $\mathcal{N}_{\mathcal{I}_d}$ and $\mathcal{N}_{\mathcal{I}_f}$ consist of, respectively, the interior nodes and triangles of the slip surface $\mathcal{I}_d$. These sets may be made of several manifold connected components, tied by radial edges [53]. When the deformed interface has
reached a new position, the set of triangles $T_s$ will have to be re-computed from the original nodes $N_s$ to account for the new contact position.

Provided that the modifiable domain is small enough (part of the regions $R_1$ and $R_2$ has to lie outside the sphere of influence), the slip surface can be parameterized by a bijective function $x : (u, v) \mapsto (x, y, z)$. This local parameterization is at the core of our method. First, it is used to maintain a geometrically sealed contact between the deformed interface $I_d$ and the slip surface $I_s$ (Section 4). Second, it is used for a fast constrained re-meshing of $I_s$ that makes it possible to update efficiently the b-rep definition (Section 5).

In practice, the parameterization is computed by a fast algorithm described in [21, 4], that propagates recursively the gradient of the parameterization from triangle to triangle by rotating along their common edges. The final value of the gradient is computed for each node of $N_s$ by blending the contribution of its neighboring triangles. When the slip surface has a complicated shape – which is the case, in geology of most salt boundaries this parameterization algorithm is not proven to yield correct results. Therefore the advanced parameterization method described in [15] could significantly improve the method. Also, the segmentation of the slip surface would make the method globally applicable.

4. CONstrained DEFORMATION

Our method is not intrinsically dependent upon a specific deformation mechanism for the interface $I_d$. Thus, any method honoring Constraints 1 and 2 is suitable. In our current implementation, whose details are given in [4], the deformation is driven by the displacement of the contact between $I_d$ and $I_s$ (Constraint 1). As the slip surface has been parameterized, a simple planar curve interpolation scheme is used [7]. Embedding this curve in space with the parametric function $x(u, v)$ thus ensures that the contact is displaced along the slip surface.

From the new position of its border, the interior of $I_s$ can be smoothly deformed. For this purpose, the Discrete Smooth Interpolation (DSI) algorithm [18, 19] is used, as it can account for other linear geometric constraints in a unified way. Thus, the interference detection mechanism required by Constraint 2 can be implemented as a DSI constraint [19]. Once again, other deformation methods such as SCODEF [3] could be used, together with classical interference detection techniques [17], provided that the contact $C_{I_s/I_d}$ is displaced within the slip surface $I_s$.

Figure 4: The sets defined in the modified domain. The simplices of the domain are classified according to their belonging to the interior or the border of the slip surface $I_s$ or of the surface to deform $I_d$.

5. TOPOLOGY UPDATE

Once the interface $I_d$ has reached the desired position, the volume model has to be updated. As stated before, Constraints 1 and 2 do not imply a constant topology of the slip surface $I_s$ through the modification. For characterizing the possible topological changes in $I_s$, we will denote 2-manifold regions of connected triangles as polygons. From this definition, it is clear (Figure 6) that the number and the definition of polygon borders may vary. When this happens, intersections between the border edges of $E_{I_s}$ and of the modified contact $C_{I_s/I_d}$ need to be inserted in or removed from the slip surface. As shown by Figure 8, the triangles around the intersecting edges must then be modified to maintain the b-rep representational validity.

For the mesh of the slip surface itself, these topological changes are taken into account by complete retriangulation of $I_s$ inside the modified domain. This retriangulation consists of the following operations (Figure 7):

1. Destroying the previous triangulation $T_s$ (Figure 7-A).

2. Computing a Constrained Delaunay Triangulation [5] of the nodes $N_s$ in the 2D parametric space. Delaunay constraints consist of all the border edges $E_{I_s} \cup C_{I_s/I_d}$ (Figure 7-B).

3. Removing the triangles between the boundary of the modified domain and the convex hull of $N_s$ (Figure 7-C).

4. Gluing the retriangulated patch to the deformed surface $I_d$ and to the exterior of the modified domain (Figure 7-D and E).

While items 1 to 3 are either straightforward or already described in the literature, item 4 appears quite awkward to realize. Indeed, the triangulated patch is by definition
Figure 6: The deformation of $I_d$ may imply drastic topological changes in the slip surface $I_s$, such as variations in the number of polygons or in their border definition.

Figure 7: Updating the slip surface $I_s$.

Figure 9: The geometrical ambiguities raised in Figure 5 can be resolved using a topological corner map [4] (left: normal view; right: exploded view).

In this paper, we have presented a method to modify interactively triangulated, non-manifold boundary representations as used in Geosciences. This method is mainly based on:

- Necessary conditions for geological validity, which can be efficiently checked, as they are based only on topological considerations. These conditions have been used in this work for defining which types of modification are allowed.

- The parameterization of a surface to preserve geometrical contacts between the interfaces during the modification, and to perform efficiently the necessary topological updates, needed to maintain representational and geological consistency.

The use of a constrained Delaunay Triangulation and the
definition of the corner map make it possible to exploit exact geometrical predicates. As compared to a method using a surface cut algorithm [6], most node insertions are avoided, yielding more robust and efficient updates.

There are several possible extensions of this work. As mentioned in Section 3, a robust parameterization and segmentation of the slip surface [15] would extend the applicability of the method, making global modifications possible. Moreover, the b-rep topological updating has been devised to be independent from the deformation mechanism. It would thus be interesting to test different surface deformations schemes that could enforce more advanced geological constraints. For instance, a modification could involve more than one model interface, and be constrained to maintain constant volume in the regions, or to honor some inferred style of geological deformation (flexural slip, simple shear, . . . [25]). As b-rep models are often used as a basis for finite element gridding, it would also be worthwhile to recompute locally volume meshes in modified areas, in a manner similar to [8].

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7. REFERENCES


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