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Abstract : In the dual formalism of kriging, the estimation Z_v is expressed as a linear combination of dual values C_α on sample points weighted by the mean covariances $\bar{\sigma}(v, \alpha)$ plus an estimate of the trend m^* . A new method involving an eigen value decomposition of the inverse kriging matrix is proposed to compute the estimation variance. The procedure avoids block kriging of the whole deposit. It is particularly adaptable to the kriging of irregular panels. The techniques have been tested on the "Minas de Leão" coal deposit (Brazil).

Key words : Geostatistics, dual kriging, coal deposit.

I - INTRODUCTION

Kriging is a basic step for all geostatistical evaluation of reserves in ore deposits. It is a preliminary phase before the simulation of exploitation of recoverable reserves at the planning stage of a new mine or during the opening new section of an operating mine. Depending on the regionalization of mining variables which are evaluated, simple or "universal" kriging may be used. This technique provides a minimum variance, unbiased, linear estimation of the mean characteristic of blocks of any given geometry. The estimator of mean grade Z_v defined on panel v , is obtained as a solution of the kriging system. It needs the inversion of the kriging matrix for each sampling and geometrical configurations.

Since Matheron (1965, 1967, 1976), much work has been directed toward more rapid algorithms used to solve the kriging system. Several packages have been published, Delfiner (1973), Sampson (1978), Journel (1978). Unique neighbourhood techniques are commonly used to speed inversion procedures. It avoids inversion of

the kriging matrix for each geometrical configuration of panels, but the inverse matrix must be saved for each neighbourhood. The algorithm is time or space consuming, but it gives the estimate and kriging variance at each point. Nevertheless, kriging of large ore deposits is cumbersome and needs computation facilities. It is usual to obtain a complex kriging after hours of computation especially for 3D estimation. In that case, there are other compelling problems (e.g. each modification of the geometry of panel imposes a new kriging system and more computation). This has diverted the attention of the geostatistician. The more common solution is to model the whole ore deposit by elementary small blocks and to krig each block, changes in geometry of panels are approximated by an appropriated contour of elementary blocks, Deraisme (1977).

The purpose of this paper is to briefly describe how the dual formalism of kriging can be used for the evaluation of ore in mineral deposits. The estimation variance is computed by a procedure involving an eigen value decomposition of the kriging matrix which gives a good approximation of the errors. The method is computationally less expensive than direct kriging for estimation of irregular panels. The mathematical model presented is applied to the problems of estimating recoverable reserves of coal, amount of barren rocks during exploitation and layer thickness in the "Minas de Leao" coal deposit (Brazil).

II - DUAL FORMALISM OF KRIGING : THE MATHEMATICAL MODEL

II - 1 - General description

The theory and application of kriging are given by Matheron (1965), Journel and Huijbregts (1978). A recent review of dual kriging is by Matheron (1982). Similar presentation of the theory have been outlined by Orfeuil (1975) and Ripley (1981).

a) Ordinary kriging (OK) : The optimal linear estimator Z^*_K of the mean grade $Z(x)$ defined on point x is the linear combination of the n data $Z_\alpha = Z(x_\alpha)$ weighted by the n kriging ponderators λ_α obtained as a solution of the kriging system; Journel (1978) (p.306). Using a matrix form notation, we have :

$$(1) \quad Z^*_K(x) = \lambda^t(x) Z_\alpha \quad \begin{array}{l} \text{t denotes matrix transposition} \\ \lambda^t = (\lambda_1, \dots, \lambda_n, \mu) \end{array}$$

$\lambda(x)$ is solution of the kriging system :

$$(2) \quad K \lambda(x) = \sigma(x) \quad (3) \quad \lambda(x) = K^{-1} \sigma(x)$$

The "kriging" variance is given by :

$$(4) \quad \sigma^2_k = \bar{\sigma}(x,x) - \lambda^t(x) \sigma(x) = \bar{\sigma}(x,x) - \sigma^t(x) K^{-1} \sigma(x)$$

b) Dual formalism (DF) : rewritten (1) with (3) and using the fact that K^{-1} is symmetric we obtain :

$$(5) \quad Z_K^*(x) = \sigma^t(x) K^{-1} Z_\alpha = \sigma^t C_\alpha = \sum_{\beta=1}^n \sigma(x, x_\beta) C^\beta + C^{n+1}$$

The column matrix C_β depends only on data point Z_α and is a solution of the linear equation : (6) $K C_\beta = Z_\alpha$

The $n+1$ coefficients C_β are defined on each data point β . They are called dual kriging values. The new estimator (5) is obviously exact by construction : $Z_K^*(x_\alpha) = Z(x_\alpha)$ (7)

c) Geometrical interpretation : If $Z(x)$ represents a grade defined in two dimensional space, (5) is the sum of n elementary surfaces (Kernel surfaces) $\sigma(x, \beta)$ centered on x_β and weighted by C^β . Note that if $\sigma(x, \beta)$ is the covariance function of a transition model with range a , $Z^*(x)$ is defined only by the sum of surfaces such that distances between x and x_α are less than the range.

d) Physical interpretation : Similarity appears between (5) and theory of potential. More precisely, at any point x , the potential field created by n electric charges q_α fixed at points x_α is given by : $\gamma(x) = \sum q_\alpha f(x, x_\alpha)$ where $f(x, x_\alpha)$ is the harmonic potential function. This analogy shows that coefficients C^β are similar to charges and autocovariance functions to potential functions.

e) Linear properties of DF : Because of the linearity of the DF interpolator, regularization of DF fields is simply expressed as the linear combination of regularized covariances weighted by the same coefficient C_α . This type of regularization is particularly useful as it corresponds to the main applications encountered in mining practice : convolution of a grade by a given weighted function p (moving average, integration, ...) :

$$Z_p^*(x) = \int Z^*(x) p(y-x) dy = \sum_{\beta=1}^n C^\beta \bar{\sigma}_p(x, x_\beta) + C^{n+1}$$

This property can be used to compute the optimal estimation of reserves inside a unit V which has a complex shape. Computationally (5) is convenient, for C_α is a row vector that can be stored once, after which it is only necessary to evaluate regularized covariance $\bar{\sigma}_p$ for each x_α by means of classical algorithms.

II - 2 - Statistical properties of C^α

Let $Z(x)$ be a stationary random function with a constant mean m and a standard covariance σ . DF values C^β defined on each sample data points β , appear as a linear combination of random variables Z_α . Therefore it is possible to compute expectation of C^β . By means of inverse matrix blocks formula, C^{n+1} can be expres-

sed as : $C^{n+1} = \frac{u^t k^- Z}{u^t k^- u}$ u is the unit vector
 k is the kriging matrix when the mean
 is known (simple kriging SK)

so : $E(C^{n+1}) = E(Z)$. Note that C^{n+1} is an estimator of the local mean grade $E(Z)$.

Taking the expectation on both sides onto (6) gives :
 $k E(C) = 0$ (8) where $E(C)$ is the n column vector $\{C^\beta\}$ $\beta \in [1, n]$.
 If k is not ill conditioned, homogeneous system (8) has always the solution $E(C) = 0$. C^β are centered random variables. It is interesting to quote the covariance between :

C^β and Z_α : $E(C^\beta Z_\alpha) = \delta_\alpha^\beta$ (Kronecker symbol) and $E(C^\alpha C^\beta) = \sigma_{\alpha\beta}^-$;
 $\alpha, \beta \in [1, n]$. Another interesting point is that, because of the linearity of (8), the expectation of $Z^\alpha(x)$ conditioned by a given event, is the linear combination of the conditional expectation C^β by the same event.

II - 3 - The estimation variance in the dual formalism of kriging

By definition, kriging minimizes mean squares error at any data point x . The estimation variance at that point, is a linear combination of λ^α weighted by $\sigma(x, \alpha)$. This simplicity is lost in DF, so it is not possible to express estimation variance by means of DF values C^β . Our intention here is to describe a new technique of approximation of the estimation variance involving an eigenvalue and eigenvector decomposition of the kriging matrix.

a) The exact solution : Ripley (1981) suggested the use of the Cholesky decomposition of K which constructs a unique lower triangular matrix L with $K = LL^t$. To evaluate (4), we use :

$$Le = \sigma(x, x_\alpha) \quad \text{then} \quad \sigma_E^2 = \sigma^2(x) - \sum_{i=1}^n e_i^2$$

exact value of the estimation variance, but it needs to save the $n(n+1)/2$ coefficients of the triangular matrix .

b) Approximation of the estimation variance : Another way is to approximate the inverse matrix K^- by a principal component decomposition. Since kriging matrix K is symmetric and supposed to be strictly positive defined, which we assume, there is no restriction in practice. Then the K spectrum is constituted by n ordered eigenvalues $\lambda_1 > \lambda_2 > \dots > \lambda_n$ with the corresponding eigenvectors e_1, \dots, e_n . A p -approximation of K is obtained by choosing the first p eigenvectors e_1, \dots, e_p . The error depends on the p order. The matrix K^- has the same decomposition except that K^- eigenvalues are $1/\lambda_n, 1/\lambda_{n-1}, \dots, 1/\lambda_1$. A p -approximation of K^- can be expressed by means of the smallest eigenvalues of K .

$$K^* = \Omega_p^t \Lambda_p \Omega_p \quad \text{with} \quad \Omega_p = (e_1, \dots, e_p)$$

Deflation algorithm together with inverse spectral techniques are used to evaluate $1/\lambda_i$ directly on matrix K . Computationally, it is convenient to save a $p \times n$ matrix of eigenvectors and the corresponding eigenvalues. Estimation variance is obtained as follows :

$$\sigma_k^2(x) = \sigma(x,x) - \sigma^t(x,\alpha) \Omega_p^t \Lambda_p^- \Omega_p \sigma(\alpha,x)$$

If λ^- is a majoration of the neglected eigenvalues in the p -approximation of K^- , the error committed on the approximation of the variance is :

$$\lambda^- (\sigma^t(x,\alpha)\sigma(x,\alpha) - \sigma^t(x,\alpha) \Omega_p^t \Omega_p \sigma(x,\alpha))$$

The mathematical model above may be applied to prediction with an unknown trend ("Universal kriging, cokriging). Further developments are given by Matheron (1982).

III - APPLICATION TO THE "MINAS DO LEAO" COAL DEPOSIT (BRAZIL)

III - 1 - Geological setting

The stratiform coal deposit of "Minas do Leao" is located in the Permó-Carboniferous formations of "Brazilian Gondwana" which is a large sedimentary formation in South Brazil. The coal is associated with grey shales, clays and sandstones (Rio Bonito Formation), Machado (1976). This autochthonous coal has resulted of the accumulation of organic matter in a lacustrine environment together with the formation of the basin.

The coal data : the deposit was sampled by 164 vertical drills on a regular $500 \times 500m^2$ grid. The surface of the coal deposit is approximately a rectangle of $8 \times 10 km^2$. This data is used to compute recoverable reserves defined on irregular productive units according to exploitation constraints (hauling, mining holding,...).

III - 2 - Structural analysis

Three regionalized variables have been used to compute variograms in the four main directions : coal thickness (CT), barren thickness (BT) and total thickness (TT). Figures 1 a-b-c show the semi-variograms.

a) Semi-variograms of coal thickness (CT) : the experimental semi-variograms for each direction computed on the coal thickness verify the hypothesis of isotropy of the mineralization, and they can be grouped together to provide a single isotropic semi-variogram $\gamma(h) = \gamma(r)$ with $r = |h|$ shown on figure 1-a. This experimental isotropic semi-variogram can be fitted to a spherical model

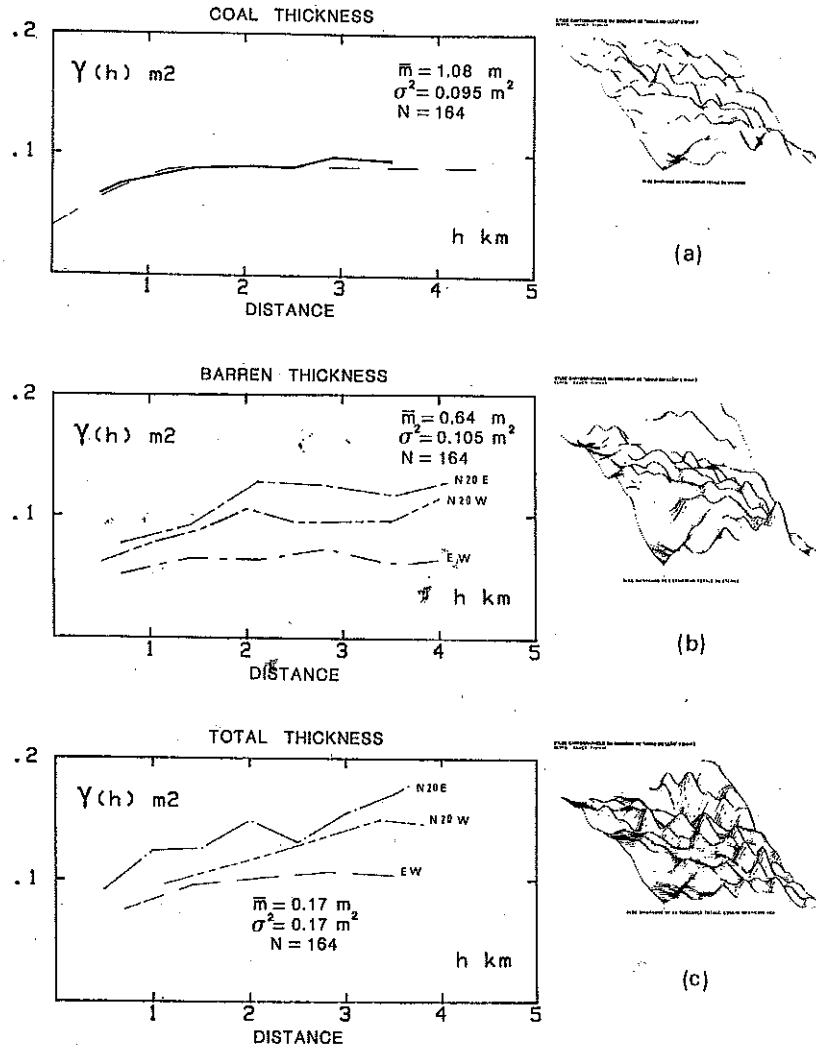


Figure 1. Regionalization of thickness in the "Miñas do Leao" coal deposit.

- a) CT isotropic semivariogram (range : $a_1 = 1500$ m)
- b) BT zonal and isotropic semivariograms (range : $a_1 = 3500$ m)
- c) TT zonal and isotropic semivariograms (ranges : $a_1 = 1500$ m, $a_2 = 3500$ m).

To the right are drawn the corresponding block diagrams for the whole deposit.

with a nugget effect : $\gamma_{CT}(r) = C_0 + \gamma_1(r) \quad \forall r \in [0, 1500 \text{ m}]$
 sill $C_1 = 0,05 \text{ m}^2$ and range $a = 1500 \text{ m}$. The nugget constant $C_0 = 0,04 \text{ m}^2$ is probably due to irregularities of interlayers at a small scale. On the hectometric scale of this observation, it is not possible to specify structures between 0 and 500 m to any greater degree of precision. However, the coal thickness at small sampling distances (10 m) was measured in the mine and variances computed on TT and CT give respectively $\sigma_{TT}^2 = 0,03 \text{ m}^2$, $\sigma_{CT}^2 = 0,017 \text{ m}^2$ indicating regionalization at a scale smaller than 500 m. The range $a = 1500 \text{ m}$ corresponds to the average sizes of sedimentary lacustrine basins.

b) Semi-variograms of barren thickness (BT) : the experimental semi-variograms for barren thickness were calculated for each of the eight directions in a plane. They represent an ideal case of a nested structure constituted by an isotropic structure with a nugget effect and an anisotropic structure depending only on the direction $D = N 20^\circ E$ of zonality (figure 1-b) :

$$\gamma_{BT}(h) = C_0 + C_1\gamma_1(r) + C_2\gamma_2(h_D)$$

where $r = |h|$; h_D is the coordinate of the vector h along the direction D of zonality. Spherical models have been adjusted with the following parameters :

nugget effect $C_0 = 0,05 \text{ m}^2$; isotropic sill $C_1 = 0,01 \text{ m}^2$; zonal sill $C_2 = 0,065 \text{ m}^2$; ranges $a_1 = 3500 \text{ m}$, $a_2 = 3500 \text{ m}$. The variability is obviously greater in the $N 20^\circ E$ direction than in the $N 135^\circ$ direction. According to orientation of productive units, quantity of barren rocks may vary considerably for the same quantity of coal. This fact was not known before the structural analysis.

c) Semi-variograms of total thickness (TT) : The experimental variograms for the total thickness ($TT = CT + BT$) are shown on figure 1-c. They can be fitted to a spherical nested structure constituted by an isotropic structure plus a nugget effect $C_0 = 0,075 \text{ m}^2$ with range $a_1 = 1500 \text{ m}$, sill $C_1 = 0,02 \text{ m}^2$, an isotropic structure with range $a_2 = 3500 \text{ m}$ sill $C_2 = 0,01 \text{ m}^2$ and a zonal structure direction $D = N 20^\circ E$ range $a_3 = 3500 \text{ m}$ sill $C_3 = 0,065 \text{ m}^2$.

$$\gamma_{TT}(h) = C_0 + C_1\gamma_1(r) + C_2\gamma_2(r) + C_3\gamma_3(h_D)$$

At a scale $< 1500 \text{ m}$, the sum of BT and CT variograms is different of the TT variograms. This means that barren thickness and coal thickness are correlated at this scale and it is particularly true for nugget effect.

The zonal anisotropy shown on variograms reflects conditions of sedimentation. Two hypotheses can be discussed :

1) barren sediments coming from Southern Bed rock erosion by rivers

2) Northern Parana Basin (marine environment) was subjected to influxes of stream sediment which have flowed over basins rich in organic matter.

Application of simple kriging to reserve evaluation

With the exploration drilling data available at 500 m spacing a classical simple kriging has been made for different drilling configurations and various estimators. Results are given in Table 1.

Central drill. poly.	Arithmetic Mean 4 pts	Kriging 9 pts	Kriging 5 pts
$\lambda_1 = 1$	$\lambda_1 = 1/4$	$\lambda_1 = .32$	$\lambda_1 = 0.33$
$\sigma_{es} = .214$	$\sigma_{es} = .12$	$\lambda_2 = .6$	$\lambda_2 = 0.67$
$\sigma_{es}/P = 20\%$	$\sigma_{es}/P = 11\%$	$\lambda_3 = .08$	$\sigma_{es} = .115$
		$\sigma_{es} = .114$	$\sigma_{es}/P = 10.6\%$
		$\sigma_{es}/P = 10.6\%$	

Table 1. Comparative results on evaluation of coal in a 500 x 500 m panel.

Because nugget effects are important and the range of main structures are about 1500 m; the five points kriging estimator gives the best estimates for recoverable reserves of coal in 500 x 500 m units. This procedure has been used to evaluate the total reserves as well as the amount of barren rock to be mined.

To be meaningful, ore reserve estimation has to reflect the mining units. For the coal mining industry, selection is not so hard as in other mineral industry because ore layers are relatively homogeneous as far as quality control is concerned (percent ash, percent sulfur, BTU,...) Armstrong et al. (1980). Hence, the whole deposit was divided into irregular productive units of about 50 x 400 m. Mining of productive units has been planned in time according to exploitation constraints (hauling, mining holding,...) in order to produce a progressive quantity of coal fixed to 250,000 t at the beginning up to 2 Mt at full rate exploitation. Modification in mining schedule may be needed during the lifetime of the mine depending on the first results of production. Therefore, simple kriging of each productive units involved the resolution of kriging system for each geometry and for each new configurations. Because the technique of saving inverse kriging matrices for each neighbourhood of drills was rather cumbersome, and computing expensive. It was decided to compute dual kriging values on each data points.

III - 3 - Application of formalism of kriging to local mineable reserve evaluation.

According to theoretical results given above, estimation of mean thickness Z_v defined on panel v is relatively easy. To give the dual kriged values C^α at each data points, one needs only the evaluation of mean covariances (or mean variograms) $\bar{\sigma}(v, x_\alpha)$ on panel v . The expression is reduced to :

$$Z_v = \sum_{\alpha=1}^n \bar{\sigma}(v, x_\alpha) C^\alpha + C^{n+1}$$

Cross validation was decided before applying it to mine design : it consisted of performing a classical ordinary kriging and comparing the values obtained to the results given by dual kriging. The $\bar{\sigma}(v, x_\alpha)$ are equal to zero for distances greater than the range. So dual kriging uses C_α only on a moving neighbourhood around the panel v , near that numerical tricks might occur. Cross validated values are given in table 2. Agreements are quite good.

Method	Quantity of coal	Estimated variance
simple kriging 9 pts	377093 t	± 39820 t
dual kriging	378895 t	± 38710 t

Table 2. Cross validated estimations obtained by ordinary kriging and by dual kriging on a 500×500 m productive unit.

Other cross validations, not given here, have been done for different geometry and different configurations.

The estimation of local mineable reserves (defined on planned mining units) for the "Minas do Leão" coal mine are given in table 3. The estimation variance was computed by means of approximations.

	Mean density of coal : 1.5		Mean density of barren : 2.5	
	1st year		2nd year	
total thickness	(1.537 \pm 0.047) m	(1.649 \pm 0.124) m		
coal thickness	(1.11 \pm 0.155) m	(0.991 \pm 0.145) m		
barren thickness	(0.427 \pm 0.161) m	(0.658 \pm 0.190) m		
surface	133 400 m ²	136 300 m ²		
coal	(222111 \pm 31016) t	(202610 \pm 29645) t		
mineable ore	(364515 \pm 36452) t	(426824 \pm 42680) t		
barren rocks	(142404 \pm 48781) t	(224214 \pm 64742) t		

Table 3. Mineable ore reserve estimation computed by means of dual kriging given for two years of production. Note the effect of zonal anisotropy on total and barren tonnage.

III - 4 - Discussion

According to the theory, whatever the sample value distribution and the geometry of productive units, the dual kriging estimator gives a good approximation of reserves. The estimating values of blocks for the first planified years of production have been obtained with a good approximation of the estimation variance. Because of zonal anisotropy, the evaluation of tonnages obtained for preplanified productive units shows :

- (i) a high variability of mineable barren rocks for the same production of coal : production of coal for the 1st and 2nd years is approximatively the same (220000 t) but mineable barren rocks vary from 360000 t to 430000 t;
- (ii) that optimization of exploitation can be performed to suspend the variability of mineable barren rocks. An appropriate combination of productive units together with new estimations based on dual kriging was decided after mining started;
- (iii) that structures at a scale less than 500 m, ought to be recognized in order to improve variograms fitting and estimators.

The "Minas do Leao" case study shows that geostatistical techniques can be used as iterative processes in order to optimize exploitation. Tools and packages can be adapted to each particular deposit even on small computers.

IV - CONCLUSIONS

Estimation of *in situ* geological resources by kriging is an important phase in any geological study. Even with ready to use packages, this step may be complex when the number of blocks to be kriged is large. It needs computer resources and is not usually performed several times. According to the theory, the dual kriging estimator is a good approximation for evaluation of mining units. The linearity of the expressions means that the technique is appropriate to small computers : after solving the linear dual system of kriging by any inverse method (for example by successive approximations Journel (1978)) dual values c^{α} computed on each data points, can be saved. Hence, estimators need only the theoretical model of covariances (or variograms) and a subroutine to compute the mean covariances. The method is computationally less expensive than direct kriging. It is recommended :

- (i) that estimation of irregular panels uses spatially distributed data;
- (ii) that dual kriging be used when the number of kriged blocks is greater than the number of data points (1000 drill holes used to krige a 3D deposit approximated by 200000 blocks);
- (iii) as a preliminary step before the simulation of ore deposits.

However, the main difficulty of the technique is that it is not possible to obtain easily the estimation variance. There are many ways of solving the problem :

(i) find an approximation of the estimation variance by means of a Cholesky triangularization of the kriging matrix;
 (ii) orthogonalize the basis spanning the n-dimensional space - R mode principal components extraction is one of the possible methods; n is the number of sample points. Dual formalism of kriging is also useful in direct contouring (see Galli this vol.). The formalism is also useful to obtain theoretical results on linear operators applied to regular grids estimated by kriging (moving average, Fast Fourier Transform, derivation, integration...) : the properties of filtered kriging fields are contained in the covariance model and in the trend.

The theory can be extended to "Universal" kriging and cokriging. Most important, special attentions are actually directed on multivariate processing and the dual kriging formalism opens new fields for application of multivariate geostatistics.

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